Complementarity and the Amplification of Financial Frictions

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Abstract

Does improving the access to financing lead to more productive firms? We study the implications of complementarity on productivity under financial frictions. We develop a model of heterogeneous producers facing collateral constraints and a technology adoption decision. Producers' choices are connected through the final good market, creating complementarity in adoption. We find that more complementarity between producers amplifies the effects of financial frictions through more sensitive adoption. In a high-complementarity economy, adopters increase up to 117% relative to the self-financing economy. Misallocation is the main driver of TFP loss only when complementarity is low. Our results imply that economies with high complementarity benefit more from financial liberalizations, incentivizing the adoption of more productive technologies.

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1 Introduction

Financial frictions not only distort capital allocation but also technology adoption, which changes the productivity distribution. The aggregate effect of the distorted distribution could be sizable if a large share of producers give up adopting more productive technologies due to lack of financing. In this case, policies to relax financial frictions bring large benefits to the economy. However, if producers choose not to adopt when financing improves, the effects of the policies can be significantly milder. The recent development literature argues that complementarity between firms' decisions increases the response of technology adoption to distortions, thus amplifying the aggregate effects from firm-level frictions.

In this paper, we focus on financial frictions as the main source of distortions. Does complementarity amplify the effect of financial frictions? How much aggregate productivity is lost due to distorted technology adoption? To answer these questions, we build a heterogeneous-agent model with financial frictions and technology adoption, introducing intermediate goods to the production of the final and adoption goods, which creates complementarity between producers. We calibrate the model to the US economy without distortions and conduct counterfactual analyses under different levels of complementarity.

We show that more complementarity between intermediate goods amplifies the effects of financial frictions. Specifically, when intermediate goods are less substitutable (more complementary), the share of adopters responds more to a relaxation of financial frictions. This results in larger aggregate effects on TFP. On the other hand, when intermediate goods are perfect substitutes, technology adoption is muted in our setting and hence all aggregate effects are mostly explained by capital misallocation. Our findings suggest economies with higher complementarity benefit more from better access to financing.

We focus on two interaction channels: intermediate good substitutability and the intensity of final goods in the production of the adoption good. First, when intermediate goods are more complementary, individual producers have more incentives to increase output through positive spillovers. This effect is likewise reflected in technology adoption: when other producers adopt more productive technologies, the value of adoption increases generating an amplification effect. The effects are larger when the economy features more complementarity. Similarly, the spillover has the opposite effect when producers are more constrained and their production level is more negatively affected by financial frictions. Secondly, we find that the effects of financial frictions on adoption, GDP, and aggregate TFP are amplified when the adoption good is more intensive in final good than labor.

Our model features a mass of intermediate good producers, who are heterogeneous in the quality of their output and their wealth. Producers have the option to adopt a more productive technology by paying a one-time adoption cost in units of intangible capital, increasing their optimal scale. Intangible capital is produced with a constant returns to scale technology that employs final good and labor. The final good producer aggregates intermediate goods using a constant elasticity of substitution technology, creating linkages between the producers' decisions.

Intermediate good producers face the standard problem in the financial frictions literature, with their capital demand subject to a collateral constraint. Producers' output therefore depends on both their productivity and their wealth. Intermediate goods are perfectly substitutable within the same sector, so producers are price takers. Intermediate goods are then aggregated into the final good, that is used for consumption, investment, and as an input in the production of the adoption good.

Producers' are initially unproductive. They have the option to pay a one-time adoption cost to use a more productive technology. We interpret the adoption cost as investment in intangible capital. Adopting the productive technology allows to increase the optimal scale of production through a jump in productivity and allows to pledge intangible capital as collateral to rent physical capital.

Related Literature The macro-development literature has widely studied misallocation of inputs, with influential work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[1] Our model is the most related to frameworks with financial frictions as the source

[1] Gopinath et al. (2017) study the increased cost of misallocation in Europe and the decline in sectoral total factor productivity in response to a decline in the real interest rate. Inspired by the analysis in Bau and Matray (2023), we focus on the access to financial markets as changes in collateral constraints and

of misallocation. Some examples with an entry or adoption margin include Midrigan and Xu (2014), Bento and Restuccia (2017), and Fattal-Jaef (2022). Buera, Kaboski and Shin (2011) has a sectoral choice that can also be interpreted as an adoption margin. Our paper introduces a continuum of sectors with heterogeneous producers and an adoption-specific good to study the amplification through prices. This specification allows for an expanded notion of sector and the analysis of amplification of distortions through complementarity in production.

The paper also relates to the literature on complementarity in technology adoption. Murphy et al. (1989) explores this idea in a model with aggregate demand spillovers.[2] Buera et al. (2021) analyze the amplification of distortions in a static model with heterogeneity. Our model focuses on the amplification of financial frictions and technology adoption through production complementarity, which introduces spillovers. We study this problem in a heterogeneous-agent dynamic framework with financial frictions as the underlying distortion, meaning that distortions arise endogenously and indirectly affect the decisions of other producers.

Our paper contributes to the literature studying the general equilibrium effects from individual distortions. Atkeson and Burstein (2010) find that changes in trade costs at the producer level can be partially offset in general equilibrium by the cost of innovation. In our model, easing financial frictions might not translate into more adoption in the absence of complementarity. We explore this channel by introducing final goods and labor as inputs in the production of the adoption good.

Our model helps explain the recent empirical evidence on misallocation in Bau and Matray (2023). They use a financial liberalization in India during the 2000s that expanded the international funding allowed for firms to study the effects of misallocation. While they find improvements in the distribution of capital and an overall reduction of misallocation, they get statistically insignificant results when analyzing changes in within-firm TFP. We rationalize this fact through the introduction of complementarity in production in a standard

distortions.

[2]In open-economy settings, Okuno-Fujiwara (1988), Rodrik (1996), and Rodríguez-Clare (1996) study coordination failures and industrialization policies. We do counterfactual analysis in an open economy setting but abstract from coordination failures. Our model instead features spillovers in adoption.

macro-development model with misallocation due to financial frictions. We find that low complementarity between producers can mute the response of technological adoption when firms' financing is expanded.

2 Model

Our model builds on Midrigan and Xu (2014). The economy is populated by producers and workers. The efficiency of labor and the measure of producers grow at the same constant rate g. Specifically, producers are born exogenously and initially operate an *unproductive* technology. Producers transform labor and capital into an intermediate good according to their type z. Within a type z, there is a continuum of producers with perfectly substitutable output. A competitive firm aggregates intermediate goods into the final good.

2.1 Sectoral Problems

2.1.1 Final Good Sector

A competitive firm produces the final good using intermediate goods. Intermediate goods are indexed by $z \in Z = \mathbb{R}_{++}$. The technology of production for the final good is

$$Y = \left[\int_Z Y(z)^{1-\frac{1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.1}$$

where $\sigma > 1$. Therefore, the final good producer demands intermediates from each type z according to the inverse demand

$$p(z) = \left[Y(z)/Y \right]^{-1/\sigma} P. \tag{2.2}$$

The final good is the numeraire in our model, so $P = \left[\int_Z p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} = 1$. By assumption, intermediate good producers of type z take $p(z)$ as given and their output is perfectly substitutable among producers with the same z. We refer to the mass of producers with productivity z as *sector* z. We now describe their problem.

2.1.2 Intermediate Good Sector

Intermediate good producers in sector z are characterized by their exogenous, stochastic productivity z, their asset holdings $a \in A = [\underline{a}, \bar{a}]$, and their technology of production. They are born exogenously at rate g and live indefinitely. The idiosyncratic quality z follows the stochastic process

$$\log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t \tag{2.3}$$

with $\rho \in (0, 1)$ and $\varepsilon_t \sim N(0, \sigma_z)$.

There are two types of technologies (or producers): productive and unproductive. All producers can employ the unproductive technology. Upon payment of a one-time adoption cost, producers earn the right to use the productive technology and we refer to them as productive producers. Producers maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to their technology-specific budget and collateral constraints.

Productive Producers The production function is

$$y^p(k, \ell) = [z \exp(\phi)]^{1-\eta_p} \left(k^{\alpha_p} \ell^{1-\alpha_p}\right)^{\eta_p}, \tag{2.4}$$

where $\eta_p \in (0, 1)$ denotes the returns to scale, and $\phi > 0$ is the productivity advantage of the productive technology. Given their quality z, market price $p(z)$, and wealth a, producers maximize their profits. They employ labor and capital at market prices W and $r + \delta$, respectively. Producers face a collateral constraint to rent capital: they can borrow capital depending on their wealth a and their intangible capital $P_\kappa \kappa$ (in units of the numeraire). The constrained profit maximization problem is

$$\Pi^p(z, a) = \max_{k,\ell} \; p(z) [z \exp(\phi)]^{1-\eta_p} k^{\alpha_p \eta_p} \ell^{(1-\alpha_p)\eta_p} - W\ell - (r + \delta)k, \tag{2.5}$$

subject to the collateral constraint

$$k \leq \frac{1}{1-\theta}a + \frac{\theta}{1-\theta}P_{\kappa}\kappa,$$

where $P_{\kappa}\kappa$ corresponds to the producer's fixed stock of adoption good, which we interpret as intangible capital.[3] We assume they can pledge intangible capital as collateral to rent physical capital up to a fraction $\theta/(1-\theta)$. In the self-financing world ($\theta = 0$), intangible capital is not pledgeable and producers can only rent up to their wealth level.

The demand for inputs is given by

$$r + \delta + \mu^p(z,a) = \alpha_p \eta_p \frac{p(z)y^p(z,a)}{k^p(z,a)},$$

and

$$W = (1-\alpha_p)\eta_p \frac{p(z)y^p(z,a)}{\ell^p(z,a)},$$

where $\mu^p(z,a) \geq 0$ is the multiplier on the collateral constraint. When $\mu^p(z,a) = 0$, producers are not constrained in their capital demand and can achieve their optimal scale. When they are constrained, $r + \delta + \mu^p(\cdot)$ will reflect the shadow return on capital the producer is facing.

[3] Upon adoption, producers need to acquire κ units of adoption good at price P_{κ}. This adoption cost is an intransferable investment made once in the producers' lifetime that gives them the right to use the productive technology.

Optimal profits are[4]

$$\begin{aligned}\Pi^p(z,a) =& p(z)y^p(z,a) - W\ell^p(z,a) - [r+\delta]\, k^p(z,a)\\ =& z\exp(\phi)p(z)^{\frac{1}{1-\eta_p}} \underbrace{\left(\frac{\alpha_p\eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p\eta_p}{1-\eta_p}}}_{①} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}}\\ &\times \underbrace{\left[1-\eta_p+\frac{\mu^p(z,a)}{r+\delta+\mu^p(z,a)}\alpha_p\eta_p\right]}_{②}. \end{aligned} \tag{2.6}$$

The excess return on capital $\mu^p(\cdot)$ has two distinct effects on profits. First, $\mu^p(\cdot)>0$ distorts the capital to output ratio within a firm. Thus, producers hire less capital than they would like to and thus output is relatively low compared to the optimal scale.[5] Second, given the labor share $(1-\alpha)\eta$, $\mu^p(\cdot)>0$ also distorts the profit share, which equals $1-\eta$ in the unconstrained case.

Given profits and interest payments from their asset holdings, they choose their consumption level c and future asset holdings a' in order to maximize their value in recursive form

$$V^p(z,a) = \max_{c,a'} \log(c) + \beta\mathbb{E}_{z'}\left[V^p(z',a')\middle|\, z\right],$$

subject to the budget constraint

$$c + a' = \Pi^p(z,a) + (1+r)a.$$

[4]Input demands are

$$k^p(z,a) = [z\exp(\phi)]\, p(z)^{\frac{1}{1-\eta_p}} \left(\frac{\alpha_p\eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p\eta_p}{1-\eta_p}+1} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}}$$

and

$$\ell^p(z,a) = [z\exp(\phi)]\, p(z)^{\frac{1}{1-\eta_p}} \left(\frac{\alpha_p\eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p\eta_p}{1-\eta_p}} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}+1}.$$

Output is

$$y^p(z,a) = [z\exp(\phi)]\, p(z)^{\frac{\eta_p}{1-\eta_p}} \left(\frac{\alpha_p\eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p\eta_p}{1-\eta_p}} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}}.$$

[5]From the first order condition with respect to capital, $\frac{\alpha_p\eta_p}{r+\delta+\mu^p(\cdot)} = \frac{k^p(\cdot)}{p(z)y^p(\cdot)}$.

Unproductive Producers Upon birth, producers can only operate a less productive or traditional technology that we call *unproductive*. They are born with quality drawn from the stationary distribution of z and wealth $a_0 = \min\{a \in A \cap \mathbb{R}_{++}\}$. The production function is

$$y^u(k,\ell) = z^{1-\eta_u}\left(k^{\alpha_u}\ell^{1-\alpha_u}\right)^{\eta_u}, \tag{2.7}$$

with $\eta_u \leq \eta_p$ and $\alpha_u \leq \alpha_p$. This parameterization implies that the unproductive technology is less intensive in capital, less profitable, and has a smaller optimal scale. Therefore, unproductive producers will have a smaller operation for the same (z,a) than productive producers. Given their quality z, market price $p(z)$, and wealth a, producers maximize their profits

$$\Pi^u(z,a) = \max_{k,\ell}\; p(z)z^{1-\eta_u}k^{\alpha_u\eta_u}\ell^{(1-\alpha_u)\eta_u} - W\ell - (r+\delta)k, \tag{2.8}$$

subject to the collateral constraint

$$k \leq \frac{1}{1-\theta}a.$$

Similar to productive producers, the unproductive producers' profits are

$$\begin{aligned}\Pi^u(z,a) &= p(z)y^u(z,a) - W\ell^u(z,a) - [r+\delta]\,k^u(z,a)\\ &= zp(z)^{\frac{1}{1-\eta_u}}\left(\frac{\alpha_u\eta_u}{r+\delta+\mu^u(z,a)}\right)^{\frac{\alpha_u\eta_u}{1-\eta_u}}\left(\frac{(1-\alpha_u)\eta_u}{W}\right)^{\frac{(1-\alpha_u)\eta_u}{1-\eta_u}}\\ &\quad\times\left[1-\eta_u+\frac{\mu^u(z,a)}{r+\delta+\mu^u(z,a)}\alpha_u\eta_u\right].\end{aligned} \tag{2.9}$$

where $\mu^u(z,a) \geq 0$ is the multiplier on the collateral constraint.

In addition to the consumption-saving decision, unproductive producers have the option to adopt the productive technology. Adoption occurs upon payment of a one-time cost κ in units of the adoption good. Therefore, their current value is

$$V^u(z,a) = \max_{c,a'} \log(c) + \beta\max\left\{\mathbb{E}_{z'}\left[V^u(z',a')|\,z\right], \mathbb{E}_{z'}\left[V^p(z',a'-P_\kappa\kappa)|\,z\right]\right\},$$

subject to the budget constraint

$$c + a' - \xi(z,a) P_\kappa \kappa = \Pi^u(z,a) + (1+r)a.$$

The indicator function $\xi(z,a)$ equals 1 when an unproductive producer decides to adopt the productive technology next period, and 0 otherwise. Since the quality process is independent, the distribution of z' conditional on z does not depend on the adoption decision.

Adoption Gain To better understand the adoption decision by producers, consider the profit gap for a given state (z,a):

$$\begin{aligned}
\log\left[\frac{\Pi^p(z,a)}{\Pi^u(z,a)}\right] = \quad & \phi - \frac{\alpha_p \eta_p}{1-\eta_p} \log\left[r + \delta + \mu^p(z,a)\right] \\
& + \frac{\alpha_u \eta_u}{1-\eta_u} \log\left[r + \delta + \mu^u(z,a)\right] \\
& + \log\left(1 - \eta_p + \frac{\mu^p(z,a)}{r + \delta + \mu^p(z,a)} \alpha_p \eta_p\right) \\
& - \log\left(1 - \eta_u + \frac{\mu^u(z,a)}{r + \delta + \mu^u(z,a)} \alpha_u \eta_u\right).
\end{aligned}$$

In the perfect-borrowing world, the log profit gap reduces to the productivity gain ϕ. However, when the producers are constrained, they also have to account for two additional effects. First, a distorted capital to output ratio reduces revenue overall and is reflected by the shadow return on capital $r + \delta + \mu^i(\cdot)$. By adopting the productive technology, producers get access to more financing by pledging their intangible capital as collateral and simultaneously increase their optimal scale. Second, a high return on capital increases the profit share of producers: since the labor share is constant, any return in excess of the market price of capital is absorbed by the producer. Since producers are price takers and output is perfectly substitutable within sector z, the elasticity of substitution σ does not directly affect the profit gap.

2.1.3 Adoption Good Sector

Producers need to acquire κ units of the adoption good in order to utilize the productive technology. In our calibration, we interpret the adoption cost as investment in intangible capital (intellectual property). The adoption good is produced by a competitive firm that combines final goods and labor according to

$$y_\kappa(x, \ell) = \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} x^\gamma \ell^{1-\gamma}.$$

Therefore, the price of the adoption good is equal to the marginal cost $P_\kappa = P^\gamma W^{1-\gamma}$. The elasticity γ indicates the intensity of the adoption good in final goods. When γ is lower, the production of the adoption good is more intensive in labor and thus more time is needed to produce the same output. The demand for inputs conditional on total demand for adoption good κM, where M is the mass of adopters, is

$$X^\kappa = \gamma P^{-(1-\gamma)} W^{1-\gamma} \kappa M \quad \text{and} \quad L^\kappa = (1-\gamma) P^\gamma W^{-\gamma} \kappa M.$$

2.1.4 Workers

There's a unit measure of workers in the economy. Each of them is subject to an idiosyncratic shock ν_t to labor efficiency that follows a finite-state Markov process. Their efficiency grows at rate g, hence their labor supply equals $g^t \nu_t$, which they supply inelastically. They consume and save in order to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to the budget constraint

$$c_t + a_{t+1} = W g^t \nu_t + (1+r) a_t.$$

Since ν_t is stochastic, workers save for precautionary motives and supply assets in equilibrium.

2.2 Aggregation

To get expressions for aggregate output and productivity, we assume $\alpha_u = \alpha_p \equiv \alpha$ and $\eta_u = \eta_p \equiv \eta$. For compactness, denote capital return as $R^i(z,a) = r + \delta + \mu^i(z,a)$.

2.2.1 Aggregation Within Sector

Given sectoral price $p(z)$, the quantity produced in sector z is

$$Y(z) = p(z)^{\frac{\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} (\alpha\eta)^{\frac{\alpha\eta}{1-\eta}} \\ \times \underbrace{\left[[z\exp(\phi)] \int_A R^p(z,a)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^p(z,a) + z \int_A R^u(z,a)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^u(z,a) \right]}_{\equiv \Lambda_y(z)}.$$

The aggregator $\Lambda_y(z)$ is a weighted average of productivities. For clarity, we can rewrite it as

$$\Lambda_y(z) \propto \exp(\phi) \int_A \left(\frac{k^p(z,a)}{y^p(z,a)}\right)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^p(z,a) + \int_A \left(\frac{k^u(z,a)}{y^u(z,a)}\right)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^u(z,a),$$

where the weights depend on the capital to output ratio and reflect the level of distortion from the optimal scale. Substituting the sectoral price $p(z) = [Y(z)/Y]^{-1/\sigma}$, we get

$$Y(z) = Y^{\frac{\eta}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta\sigma}{(1-\eta)\sigma+\eta}} (\alpha\eta)^{\frac{\alpha\eta\sigma}{(1-\eta)\sigma+\eta}} \Lambda_y(z)^{\frac{(1-\eta)\sigma}{(1-\eta)\sigma+\eta}}.$$

When σ is lower, varieties are less substitutable and the quantity produced $Y(z)$ depends more on the aggregate quantity Y. In particular, $d\log Y(z)/[d\log Y] \to \eta$ as $\sigma \to 1$ (more complementarity), and $d\log Y(z)/[d\log Y] \to 0$ as $\sigma \to \infty$ (no complementarity). The elasticity of substitution therefore governs the interconnections between sectors, implying that complementarity (low σ) creates positive spillovers between varieties. Similarly, input demands in the sector are

$$\begin{aligned}
K(z) &= \int_A k^p(z,a)d\Omega^p(z,a) + \int_A k^u(z,a)d\Omega^u(z,a) \\
&= Y^{\frac{1}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta(\sigma-1)}{(1-\eta)\sigma+\eta}} (\alpha\eta)^{\frac{\alpha\eta(\sigma-1)}{(1-\eta)\sigma+\eta}+1} \Lambda_y(z)^{-\frac{1}{(1-\eta)\sigma+\eta}} \\
&\quad \times \underbrace{\left[[z\exp(\phi)] \int_A R^p(z,a)^{-\frac{\alpha\eta}{1-\eta}-1} d\Omega^p(z,a) + z\int_A R^u(z,a)^{-\frac{\alpha\eta}{1-\eta}-1} d\Omega^u(z,a)\right]}_{\equiv \Lambda_k(z)},
\end{aligned}$$

and

$$\begin{aligned}
L(z) &= \int_A \ell^p(z,a)d\Omega^p(z,a) + \int_A \ell^u(z,a)d\Omega^u(z,a) \\
&= Y^{\frac{1}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta(\sigma-1)}{(1-\eta)\sigma+\eta}+1} (\alpha\eta)^{\frac{\alpha\eta(\sigma-1)}{(1-\eta)\sigma+\eta}} \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}}.
\end{aligned}$$

Let $\mathcal{A}(z) \equiv Y(z)\left[K(z)^\alpha L(z)^{1-\alpha}\right]^{-\eta}$ denote TFP in sector z. Then,

$$\mathcal{A}(z) = \frac{\Lambda_y(z)^{1-(1-\alpha)\eta}}{\Lambda_k(z)^{\alpha\eta}}.$$

This expression for TFP is similar to Midrigan and Xu (2014), where outputs are perfect substitutes. Therefore, variety substitutability does not directly affect productivity within a sector as producers face the same price and z is exogenous. Despite no direct effect, substitutability has an indirect effect on sectoral TFP $\mathcal{A}(z)$ through capital returns and the marginal distribution over wealth. This fact highlights the role of the interaction between collateral constraints and variety substitutability, resulting in more amplification of financial frictions.

2.2.2 Aggregation Across Sectors

To compute aggregate productivity, let $K = \int_Z K(z)dz$ and $L = \int_Z L(z)dz$. Aggregate output is

$$Y = \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} (\alpha\eta)^{\frac{\alpha\eta}{1-\eta}} \left[\int_Z \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{(1-\eta)\sigma+\eta}{(1-\eta)(\sigma-1)}}.$$

We can then get an expression for the sectoral price:

$$p(z) = \left[\int_Z \left(\frac{\Lambda_y(z)}{\Lambda_y(\tilde{z})}\right)^{-\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} d\tilde{z}\right]^{\frac{1}{\sigma-1}}$$

The sectoral price $p(z)$ relates sectoral efficiency: when a sector is relatively more efficient than the others (lower $\Lambda_y(z)$), its price is lower.

Define $\mathcal{A} \equiv Y\left[K^\alpha L^{1-\alpha}\right]^{-\eta}$, then

$$\mathcal{A} = \frac{\left[\int_Z \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{\sigma}{\sigma-1}-(1-\alpha)\eta}}{\left[\int_Z \Lambda_y(z)^{-\frac{1}{(1-\eta)\sigma+\eta}} \Lambda_k(z)dz\right]^{\alpha\eta}}.$$

Since we want to measure the effect of distortions on aggregate TFP, define average productivity in sector z as

$$\Lambda^e(z) = z\left[\exp(\phi)\int_A d\Omega^p(z,a) + \int_A d\Omega^u(z,a)\right].$$

Average productivity is naturally increasing in the number of adopters within z: when all producers adopt, $\Lambda^e(z) = z\exp(\phi)$ times the mass of producers in z; when none does, $\Lambda^e(z) = z$ times the mass of producers in z. Under no distortions ($\mu^i(a,z) = 0$), $\Lambda_k(z) = (r+\delta)^{-1}\Lambda_y(z) = (r+\delta)^{-\frac{\alpha\eta}{1-\eta}-1}\Lambda^e(z)$. Similarly, $\mathcal{A}^e(z) = \Lambda^e(z)^{1-\eta}$. The undistorted aggregate productivity is

$$\mathcal{A}^e = \left[\int_Z \mathcal{A}^e(z)^{\frac{\sigma-1}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{(1-\eta)\sigma+\eta}{\sigma-1}} \tag{2.10}$$

We interpret $\log \mathcal{A}^e - \log \mathcal{A}$ as the loss from misallocation. That is, for a fixed measure $\Omega^i(z,a)$, $i = \{u,p\}$, the productivity gain of reallocating capital across producers.

2.3 Equilibrium Conditions

Let $\Omega_t^i(z,a)$ denote the measure of producers with quality z and wealth a using technology i at time t. Denote $f(z'|z)$ the conditional distribution of quality and $\bar{f}(z)$ the stationary distribution over Z. The measure of productive producers evolves over time according to

$$\Omega_{t+1}^p(z',A) = \int_A \int_Z f(z'|z) d\Omega_t^p(z,a) + \int_A \int_Z f(z'|z)\xi(z,a) d\Omega_t^u(z,a).$$

The measure accounts for producers that were productive in the previous period and those who adopted after t. The measure of unproductive producers is

$$\Omega_{t+1}^u(z',A) = \int_A \int_Z f(z'|z) \left[1 - \xi(z,a)\right] d\Omega_t^u(z,a) + (g-1)\bar{f}(z) N_t,$$

where $N_t = g^t$ is the total number of producers at t. We normalize the mean of ν_t, the idiosyncratic efficiency of workers, so that $L_t = g^t$. Each period, the mass of adopters is

$$M_t = \int_{Z\times A} \xi(z,a) d\Omega_t^u(z,a).$$

The quantity produced in sector z is

$$Y_t(z) = \sum_{i=u,p} \int_A y^i(z,a) d\Omega_t^i(z,a).$$

Output of final good is

$$\begin{aligned} Y_t &= \left[\int_Z Y_t(z)^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\int_Z \left(\int_A y^p(z,a) d\Omega_t^p(z,a) + \int_A y^u(z,a) d\Omega_t^u(z,a)\right)^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \end{aligned}$$

The market clearing conditions for labor and assets are

$$\begin{aligned} L_t &= L_t^p + L_t^u + L_t^\kappa \\ &= \sum_{i=u,p} \int_{Z\times A} \ell^i(z,a) d\Omega_t^i(a,z) + (1-\gamma) P^\gamma W^{-\gamma} \kappa M_t, \end{aligned} \tag{2.11}$$

$$\begin{aligned} 0 &= A_t^w + A_t^p + A_t^u - (K_t^p + K_t^u) \\ &= A_t^w + \sum_{i=u,p} \int_{Z\times A} a^i(z,a) d\Omega_t^i(a,z) - \sum_{i=u,p} \int_{Z\times A} k^i(z,a) d\Omega_t^i(a,z). \end{aligned} \tag{2.12}$$

A competitive equilibrium in this economy consists of prices $\{\{p(z) : z \in Z\}, W, r\}$ and measures $\Omega_t^i(z,a)$ such that all agents maximize their problems given prices and all markets clear, i.e., $Y_t(z) = p(z)^{-\sigma} Y_t$ for all $z \in Z$, and equations 2.11 and 2.12 hold with equality.

3 Quantitative Analysis

In this section, we conduct counterfactual analysis to understand the interaction between technology adoption and financial frictions, and how complementarity across sectors affects this interaction. We first calibrate our model to the US economy under perfect borrowing, yielding a benchmark without distortions. We then evaluate the model featuring low and high complementarity in production, for different degrees of financial frictions.

3.1 Calibration

We calibrate the model to the US economy as an undistorted benchmark ($\theta = 1$) in a closed economy setting. We assign standard values to several parameters, following mostly Midrigan and Xu (2014). For the production function, we choose capital elasticity $\alpha = 0.33$ and span of control $\eta = 0.85$, both common across technologies. Capital depreciation is 0.06. We fix the growth rate g to 2 percent, which in the model equals the entry rate of intermediate good producers and the growth rate of per-worker GDP. For the idiosyncratic shock of workers, we use a two-state Markov process with $\nu_i \in \{0, 1\}$. We can interpret these values as unemployment and employment from the perspective of the worker. We set the probability of staying in the low state to 0.5, and pick the probability of staying in the high state such that employment equals 60% of population as reported by the Bureau of Labor Statistics. The mean of ν is normalized so that the total number of efficiency units of labor supplied equals the total number of producers. For the quality process, we pick values in line with those in Asker et al. (2014) for the productivity process in the US. Specifically, we choose $\rho = 0.9$ and $\sigma_z = 0.75$, making the process fairly persistent.

We choose the elasticity of substitution $\sigma = 5$ for the benchmark, as this parameter governs the interactions between sectors.[6] We use $\gamma = 1$, meaning that intangible capital is only produced with final good. We then compare to the case of $\gamma = 0$, where intangible capital is produced only using labor. For the productivity gain $(1 - \eta)\phi$, we rely on the estimations by Buera et al. (2021) and set it to 0.55.[7] We calibrate the adoption cost κ to match the ratio of investment in intangible capital to GDP, which equals around 4 percent according to the BEA's National Income and Product Accounts between 2000 and 2019. We specifically interpret private investment in intellectual property as investment in intangible capital. The resulting adoption cost κ equals 11.93, about 2.04 times aggregate output. The subjective discount factor β is calibrated to match the 2 percent interest rate in a closed-economy equilibrium.

[6] Hsieh and Klenow (2009) and Buera et al. (2021) use $\sigma = 3$ for their benchmarks, but in these models aggregation is across all differentiated good producers. Since we compare our benchmark economy to one with perfect substitution ($\sigma = \infty$), choosing $\sigma = 5$ is more conservative. All results hold qualitatively for the case of $\sigma = 3$.

[7] In particular, we interpret their productivity gap as labor productivity gap. We therefore scale it with the output-labor elasticity, i.e., $(1 - \eta)\phi = -(1 - \alpha)\log(0.43)$.

Table 1: Parameter Values

Description	Parameter	Value	Target / Source
Collateral constraint	θ	1.00	Undistorted
Elasticity of substitution	σ	5.00	Benchmark
Goods elasticity in adoption	γ	1.00	Benchmark
Discount factor	β/g	0.985	2% interest rate
Relative efficiency productive	$(1-\eta)\phi$	0.55	Buera et al. (2021)
Adoption cost	κ	11.9	Intangible investment to GDP
Span of control	η	0.85	Standard
Capital elasticity	α	0.33	Standard
Capital depreciation	δ	0.06	Standard
Growth rate	g	1.02	GDP growth & Entry
Quality Persistence	ρ	0.90	Asker et al. (2014)
Quality SD	σ_z	0.75	Asker et al. (2014)
Prob. remaining unemployed	–	0.50	Standard
Prob. remaining employed	–	0.67	Employment to population ratio

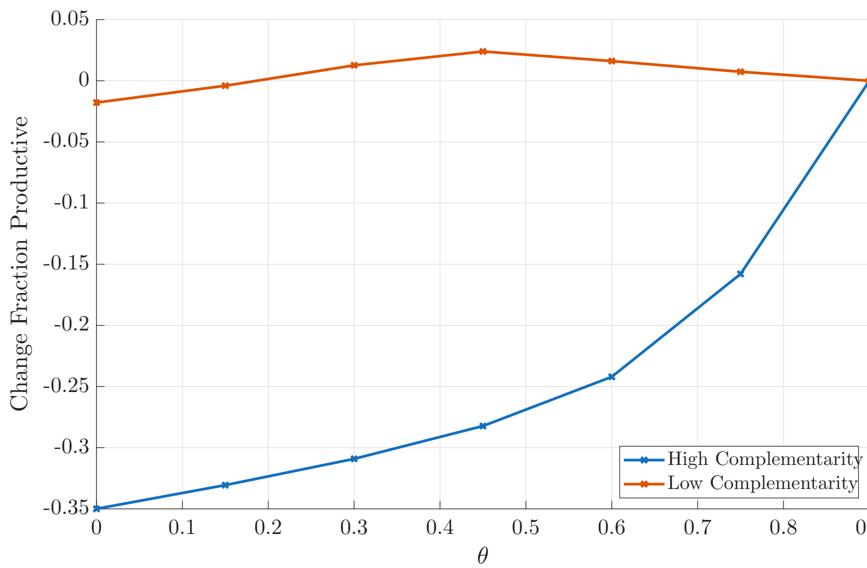
Table 2: Benchmark Results

Variable	Value
Output	5.849
Capital to Output	3.506
Consumption to Output	0.679
Tangible Investment to Output	0.281
Intangible Investment to Output	0.040
Capital to Assets	3.261
Fraction Adopters	0.980
TFP	2.485

3.2 Amplification Through Complementarity

We next analyze the interaction between complementarities ($1/\sigma$) and financial frictions (θ). In order to abstract from price changes that affect intertemporal choices and focus on the ability to borrow, the counterfactual economies take the interest rate r as given (small open economy). We set the interest rate equal to the benchmark. We compare the high complementarity case of $\sigma = 5$ and the no complementarity case of $\sigma \to \infty$ (perfect substitutes). When $\theta = 1$, producers are never constrained (perfect borrowing) and can always achieve their optimal scale. When $\theta = 0$, producers are in a self-financing economy and can demand capital up to their wealth level. Figure 1 shows the response of adoption in both economies to changes in the collateral constraint parameter θ.

Figure 1: Amplification with Complementarity

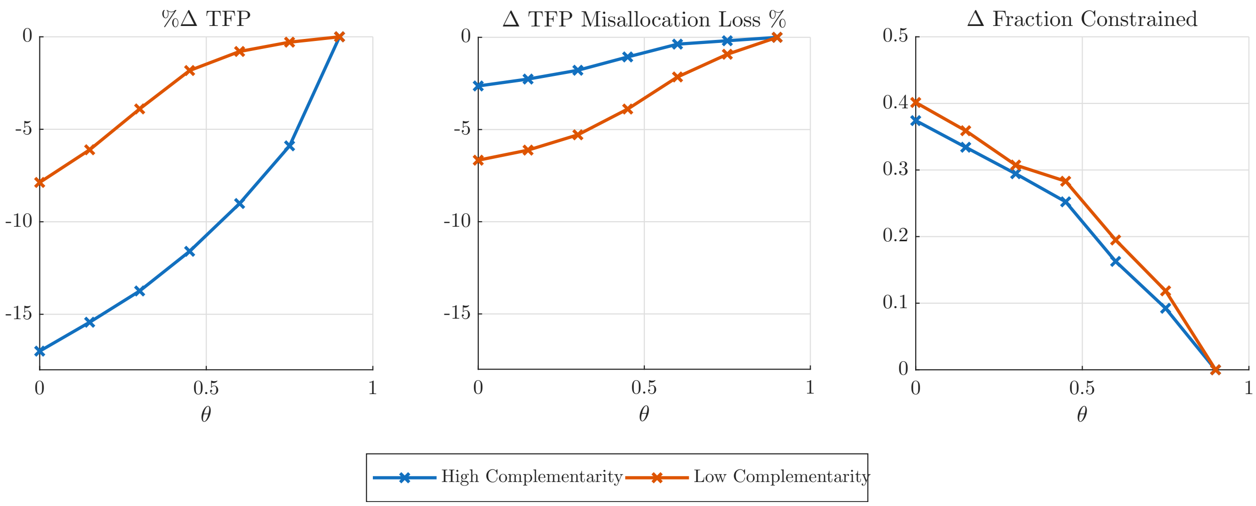


Note: Change in fraction of adopters relative to high θ case.

The economy with high complementarity experiences a large decline in the fraction of productive producers when the financial frictions increase (lower θ). On the contrary, there's a small change in the adoption rate in the no complementarity economy when financial frictions increase. This reaction implies a small change in firm-level productivity in response to a financial liberalization when there is no complementarity. The larger response of adoption in the high complementarity case translates into a larger decline of aggregate productivity compared to the no complementarity case, as shown in Figure 2.

The analysis of Figures 1 and 2 suggests that complementarity changes how financial frictions distort aggregate productivity. While most of the TFP loss comes from reduced adoption (extensive margin) in the high complementarity case, the opposite is true when there's no complementarity: almost all the TFP loss comes from misallocation (intensive margin). This result follows from the slightly higher number of constrained producers in the no complementarity economy when borrowing is harder. In the self-financing economy,

Figure 2: Aggregate Productivity and Misallocation



Note: Δ denotes change, %Δ denotes percent change. All changes relative to high θ case.

misallocation explains 15% of TFP loss in the high complementarity economy and 84% in the no complementarity economy.

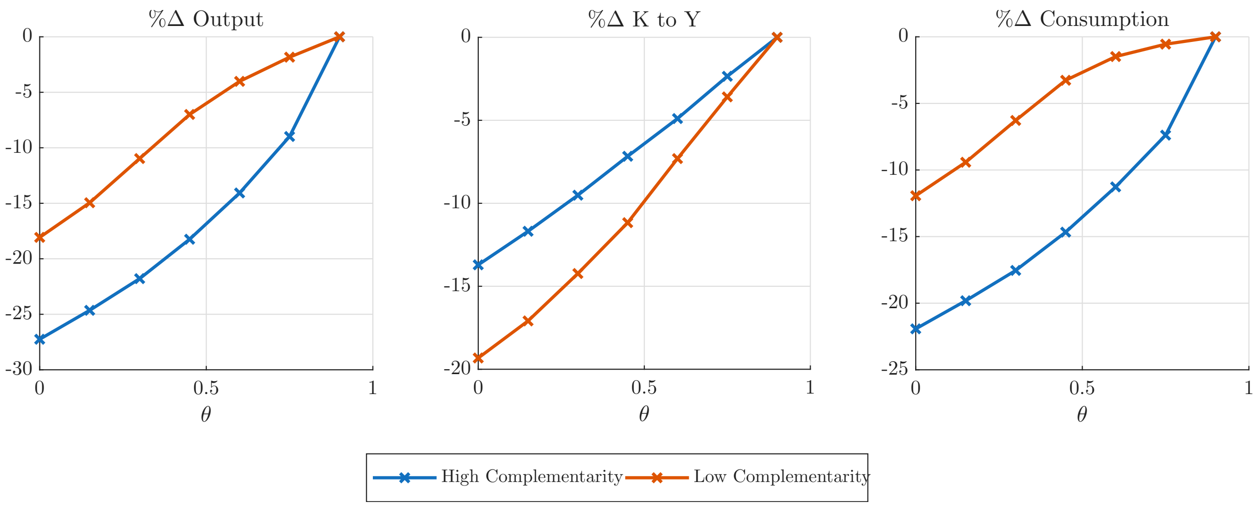
Figure 3 shows the response of GDP, capital-output ratio, and aggregate consumption for both cases. Financial frictions are more costly in terms of aggregate consumption and output when complementarity is high, both declining 10 percent more in the self-financing world relative to $\theta = 0.9$. Therefore, more complementarity amplifies the effects of financial frictions in aggregate variables.

The quantitative results of our model help explain why financial liberalizations might have small or no effects on within-firm TFP. When we consider the policy change in India during the 2000s, Bau and Matray (2023) find no significant change in TFP at the firm level and attribute most of the improvement in aggregate TFP to a reduction in capital misallocation. We interpret these results as occurring in a low complementarity environment, where weak production networks reduce the gains from adoption for producers.

In Figure 4, we compare the results in Figure 1 with two additional cases of the low complementarity (perfect substitutes) economy. We evaluate the effect of quality persistence ρ and intensity of capital between technologies α_u.[8] Both a lower persistence of quality

[8]When the technology of production does not employ capital, unproductive producers are never con-

Figure 3: Aggregate Variables



Note: %Δ denotes percent change relative to high θ case.

z and a lower intensity of capital in the unproductive technology increase the response of adoption to changes in the collateral constraint. A higher persistence decreases the incentives to adopt for low quality producers because the future stream of profits is low relative to a low persistence process.

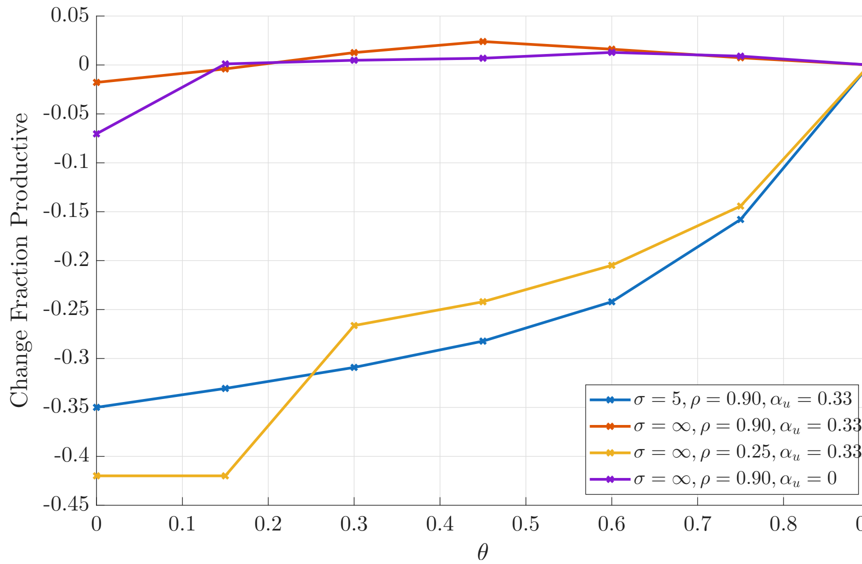
3.3 Amplification Through Intangible Investment

We now consider the role of intangible investment as a driver of amplification. As opposed to the complementarity channel we analyzed in the previous section, the technology of production for intangibles affects individual decisions solely through prices as an equilibrium effect. We focus on the two boundary cases: intangibles are produced either with final good ($\gamma = 1$) or with labor ($\gamma = 0$). We compare for both cases an economy with $\theta = 0.8$ and its self-financing counterpart of $\theta = 0$.

When intangibles are produced with final good, more adoption increases both demand and supply for goods, and thus its partial effect on incumbents is not direct. To a certain extent, there are positive spillovers (amplification) in this case even when the overall effect

strained. This specification is comparable to the traditional technology in Midrigan and Xu (2014).

Figure 4: Robustness



Note: Change in fraction of adopters relative to high θ case.

is negative and there are no complementarities between producers. When intangibles are produced only with labor, more adoption increases the demand for labor from intermediate good producers and from the intangible capital producer. Given the fixed labor supply, an increase in the number of adopters unequivocally increases the wage in the economy, lowering the production of incumbents and therefore profits.

Table 3: Amplification through Intangible Investment

	Adoption in Goods, $\gamma = 1$		Adoption in Labor, $\gamma = 0$	
	Partial Financing	Self Financing	Partial Financing	Self Financing
Fraction Adopters	0.361	0.148	0.171	0.078
Fraction Constrained	0.450	0.823	0.437	0.803
K to Y	2.126	1.886	2.130	1.889
Consumption	1.000	0.879	0.852	0.786
Output	1.188	0.920	0.998	0.814
Intangible Investment to Y	0.019	0.010	0.029	0.013
TFP	1.000	0.861	0.907	0.798
Loss Misallocation %	1.5	4.4	1.1	4.2

Note: Consumption and Output are normalized to units of consumption in Column 1, the case with adoption in goods and partial financing. TFP is normalized to 1 for the same case.

Table 3 reports the main results of the exercise. Adoption is higher in the economy where intangibles are produced with final good and it's also more affected by financing: adoption is almost 60% lower in the self-financing case when $\gamma = 1$ compared to almost 55% lower when $\gamma = 0$. This result translates into a larger difference in output: 23% lower output in self-financing for adoption in goods versus 19% for adoption in labor. Aggregate TFP in the self-financing economy is 86% of the partial economy case when $\gamma = 1$ and 88% when $\gamma = 0$. The effects of the intangible production technology on all other aggregate variables are small. Therefore, the production of intangibles has larger positive spillovers when it's more intensive in goods rather than labor.

4 Concluding Remarks

We show that complementarity greatly affects how gains from adoption change with a relaxation of financial frictions. More complementarity amplifies the effects of financial frictions in the economy, implying that the gains from better access to financial markets increase with complementarity. Most importantly, relaxing financial constraints improves technology adoption relatively more when complementarity is higher between producers.

Our model features a heterogeneous mass of producers who have the option to adopt a more productive technology by paying a one-time adoption cost in units of intangible capital, increasing their optimal scale. Their capital demand is subject to a collateral constraint, and they can rent against their assets and intangible capital. Competition between producers is perfect within their quality and demand has a constant elasticity of substitution between qualities.

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